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Note On Spectral Resolution

This note compares the transfer processes involved in three spectrally dispersing systems, with emphasis on the definition of resolution in each case. The three systems are respectively the prism, interferometric spectroscopy[1], and the multifilter method[2].

For the prism, ignoring diffraction, an incident plane wave represented by the input function of wavelength $f(\lambda)$ is mapped into an output function of angle β given by

$$g(\beta) = \int K(\lambda,\beta)\, f(\lambda)\, d\lambda$$

where the kernel K represents the transfer function of the prism. The prism thus maps the λ space into the β space. The spectral dispersion here may be arrived at by using two delta functions as input functions, $\delta(\lambda-\lambda_1)$ and $\delta(\lambda-\lambda_2)$; the output will then consist correspondingly of $\delta(\beta-\beta_1)$ and $\delta(\beta-\beta_2)$, and dispersion is given by

$$\frac{\beta_2-\beta_1}{\lambda_2-\lambda_1} = \frac{\Delta\beta}{\Delta\lambda}$$

It depends of course on the specific kernel K. Since K for the prism gives a one-to-one correspondence between λ and β, it is not necessary to transform inversely to find $f(\lambda)$.

If diffraction is now assumed present in K, K does not map delta functions into delta functions but into broadened functions, (although the mapping is still closely one-to-one) and the usual definition of resolution is arrived at by considering the dispersion coupled with the diffraction.

In interferometric spectroscopy, an input function f (λ) is mapped into an output function of path difference g (x), according to

$$g(x) = \int K(\lambda, x)\, f(\lambda)\, d\lambda$$

Here K does not merely perform a one-to-one mapping, but is more complex, being the cosine function cos $2\pi x/\lambda$. This results in two essential differences from the prism case. It is not possible to define dispersion simply, and it is necessary to make the inverse transform. Since the range of x involved in the integration of the inverse transform is necessarily finite, analogous to the finite aperture of the prism, "diffraction" appears again and limits the resolution obtained. One may consider that the direct transform here produces both dispersion and diffraction, while the inverse transform produces further diffraction.

In the multifilter method of spectral analysis, ideally an input function f (λ) is mapped by a transmission function K (λ, t) into an output function g (t), where it is a continuous variable which for example may be time:

$$g(t) = \int K(\lambda, t)\, f(\lambda)\, dt$$

In practice, the integral equation is replaced by a matrix equation since K (λ, t) is constructed from a finite number of filters. The resolution-determining factors present in the interferometer method are both present here - i.e. the nature of K and the inverse transformation - and are accompanied by a third factor, the discreteness of the system.

It would seem that a general definition of spectral resolution should take into account the three systems. It might be arrived at by the operational approach mentioned in connection with the prism, i.e. using two delta functions as input functions and finding the smallest separation between them that the given system will resolve. Thus the direct transform of a system will give, corresponding to two delta functions $\delta(\lambda-\lambda_1)$ and $\delta(\lambda-\lambda_2)$

$$\int K(\lambda, t)\,\delta(\lambda-\lambda_1)\,dt = g_1(t)$$

$$\int K(\lambda, t)\,\delta(\lambda-\lambda_2)\,dt = g_2(t)$$

If the inverse transforms of $g_1(t)$ and $g_2(t)$ are denoted by $g_1'(\lambda)$ and $g_2'(\lambda)$, their correlation function

$$\int_{-\infty}^{\infty} g_1'(\lambda)\, g_2'(\lambda)\, d\lambda$$

may be a useful measure of the congruence between the two system outputs and therefore of the resolution.

To illustrate how the Rayleigh criterion of resolution may be carried over to the general case by means of the correlation function, the prism and interferometric systems can be treated.

The output function of the prism for an input delta function $\delta(\lambda-\lambda_1)$ is, if its aperture is d,

$$\left[\frac{\sin(2\pi d \sin\theta/\lambda)}{2\pi d \sin\theta/\lambda}\right]^2 = \frac{\sin^2 x}{x^2} = f(x), \qquad (x = 2\pi d \sin\theta/\lambda)$$

The correlation function is given by[3]

$$\phi(x) = 2\pi \int_{-\infty}^{\infty} E(y)\, e^{ixy}\, dy$$

where E(y) is the square of the Fourier transform of f(x), a triangular function here:

$$E(y) = \begin{cases} \left[\frac{1}{2}\left(1-\frac{y}{2}\right)\right]^2 & 0 < y < 2 \\ \left[\frac{1}{2}\left(1+\frac{y}{2}\right)\right]^2 & -2 < y < 0 \\ 0 & \left\{\begin{array}{c} -\infty < y \leq -2 \\ 2 \leq y < \infty \end{array}\right\} \end{cases}$$

The resulting correlation function is

$$\phi_p(x) = \frac{2\pi(2x - \sin 2x)}{4x^3}.$$

For the Rayleigh separation $\theta_R = \frac{\lambda}{2d}$, ϕ has the value

$$\phi_p(x_R) \cong \frac{1}{\pi}$$

This may be normalized relative to the value of $\phi_p(0) = \frac{2\pi}{3}$, giving

$$\phi_p(\theta_R)/\phi_p(0) \cong 3/2\pi^2 \cong 0.152$$

Two wavelengths will be resolved when their diffraction patterns have a correlation less than this value.

The interferometric system results, for an input delta function $\delta(\lambda - \lambda_1)$, in the broadening function[1]

$$f(x) = \sin x / x \quad , \qquad (x = 2\pi l(\frac{1}{\lambda_1} - \frac{1}{\lambda}))$$

l being the path difference.

The normalized correlation function is

$$\phi_i(x)/\phi_i(0) = \sin x / x$$

which has the value 0.152, corresponding to the value found for the correlation of the Rayleigh separation in the prism system for $x \cong 0.95\pi$ so that

$$\frac{1}{\lambda_1} - \frac{1}{\lambda} = \frac{0.95}{2l} = \frac{\lambda - \lambda_1}{\lambda^2}$$

approximately, giving for the resolving power

$$\frac{\lambda}{\lambda-\lambda_1} = \frac{2l}{0.95\lambda}$$

In this case ~~an actual~~ a resolving power is arrived at because the "dispersion" of the system is essentially known.

The application of this definition to the multifilter system is more difficult, but would yield a quantity for a given transmission matrix which could be compared with the Rayleigh definition in the above way.

The Rayleigh criterion is used here only for purposes of comparison. It may not be the best one from the point of view of correlation, since output functions of various systems will in general bear little resemblance to the diffraction function, on which the Rayleigh criterion was based.

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   Gebbie and Vanasse - Nature 178, 432, (1956)

2. G. Wyszecki - J.O.S.A. 50, 992, (1960)

3. S. Goldman - Information theory (Prentice-Hall), 1953